NONLINEAR DEVELOPMENT OF LONGWAVE INVISCID PERTURBATIONS IN A BOUNDARY LAYER

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In a number of cases, the analysis of the propagation of small two-dimensional perturbations in a boundary layer reduces to the solution of a single nonlinear equation relative to a certain function dependent on the time and the longitudinal coordinate [1]. If the amplitude δ and the wavelength λ of the perturbations satisfy the conditions $Re^{-1/8} \ll \delta \ll 1$, $\lambda = O(Re^{-1/2}\delta^{-1})$, where the Reynolds number $Re \to \infty$ is determined on the basis of the characteristic dimension of the body in the flow, then a two-dimensional flow field in the boundary layer can be constructed by solving the Burgers equation [2] for a supersonic flow regime and the Benjamin–Ono equation [3, 4] for subsonic velocities of the incoming flow. These equations, introduced in [1] by means of asymptotic expansions of the solutions of the complete system of Navier–Stokes equations, were regarded in [5] as a consequence of a limiting transition to high-frequency large-scale perturbations in the theory of free interaction [6-8].

1. We will illustrate the derivation of the analogous one-dimensional equation by using the example of a perturbed flow in a plane jet of an incompressible fluid bounded by a solid wall [9, 10]. We will consider the time t, Cartesian coordinates x and y, velocity-vector components u and v, and pressure p to be dimensionless relative to the quantities L^*U^{*-1}, L^*, U^*, ρ^*U^{*2}, respectively (L^* and U^* are the characteristic length and characteristic velocity of the jet, ρ^* is the density of the incompressible fluid). At large $Re = U^*L^*/\nu^*$ (ν^* is the kinematic viscosity), the wall jet is analogous to a boundary layer, while the unperturbed profile U_0 of the longitudinal velocity component in the jet is dependent on the variable $Y_m = Re^{1/2}y$. The subsequent analysis is based on properties of the function U_0 which follow from the type of motion being studied. Specifically, at the outlet of the jet (with an increase in Y_m) and near the solid surface $Y_m = 0$ which bounds the jet from below,

$$U_0 \to 0,\ dU_0/dY_m \to 0,\quad Y_m \to \infty,$$
$$U_0 \to \lambda_1 Y_m + \lambda_2 Y_m^2 + \ldots,\quad Y_m \to 0. \tag{1.1}$$

Let there be a perturbation of the longitudinal velocity component of the order δ in the flow. Such a perturbation can be created, for example by placing a small obstacle of a height on the order of $y = O(Re^{-1/2}\delta)$ on the surface being traversed by the flow. We will use λ to denote the characteristic scale of the length of the perturbation wave. Then the continuity equation gives us the order of the perturbation of the vertical velocity component $v = O(Re^{-1/2}\delta\lambda^{-1})$ in the main mass of the jet at $Y_m = O(1)$. Equating the orders of the terms of the conservation equation to the vertical component of the momentum $\partial p/\partial y \sim U_0\partial v/\partial x \sim Re^{-1/2}\delta\lambda^{-2}$, we can easily evaluate the pressure perturbation $\Delta p = O(Re^{-1}\delta\lambda^{-2})$. On the other hand, near the wall, where the perturbation of velocity u is of the same order as the velocity itself, the conservation equation for the longitudinal component of momentum gives $\partial p/\partial x \sim u\partial u/\partial x$, i.e., $\Delta p = O(\delta^2)$. Comparing both estimates for Δp, we obtain the order of the wavelength of the perturbation $\lambda = O(Re^{-1/2}\delta^{-1/2})$. These estimates make it possible to represent the solution of the Navier–Stokes equations in the main body of the jet in the form

$$u = U_0 + \delta u_{1m} + \delta^2 u_{2m} + \ldots,\quad v = \delta^{3/2} v_{1m} + \delta^{5/2} v_{2m} + \ldots,$$
$$p = p_\infty + \delta^2 p_{1m} + \delta^3 p_{2m} + \ldots \tag{1.2}$$

Here, all of the sought perturbing functions with the indices 1m and 2m depend on the variables $T = Re^{1/2}\delta^{3/2}t$, $X = Re^{1/2}\delta^{1/2}x$, $Y_m = Re^{1/2}y$, while p_∞ is the pressure on the upper boundary of the jet. The system of equations of the first approximation

$$U_0\frac{\partial u_{1m}}{\partial X} + v_{1m}\frac{dU_0}{dY_m} = 0,\quad U_0\frac{\partial v_{1m}}{\partial X} = -\frac{\partial p_{1m}}{\partial Y_m},\quad \frac{\partial u_{1m}}{\partial X} + \frac{\partial v_{1m}}{\partial Y_m} = 0$$

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leads to the explicit expressions [9, 10]

$$u_{1m}=A_1(T,X)\frac{dU_0}{dY_m},\ v_{1m}=-\frac{\partial A_1}{\partial X}U_0(Y_m),\ p_{1m}=\frac{\partial^2 A_1}{\partial X^2}\int\limits_{\infty}^{Y_m}U_0^2dY'_m. \tag{1.3}$$

It can be seen from (1.1) and (1.3) that $v_{1m} \to 0$ at $Y_m \to 0$, which indicates the need to consider the following terms of the expansion for v near the wall. Inserting (1.3) into the system of equations of the second approximation

$$\frac{\partial u_{1m}}{\partial T}+U_0\frac{\partial u_{2m}}{\partial X}+u_{1m}\frac{\partial u_{1m}}{\partial X}+v_{2m}\frac{dU_0}{dY_m}+v_{1m}\frac{\partial u_{1m}}{\partial Y_m}=-\frac{\partial p_{1m}}{\partial X},$$

$$\frac{\partial v_{1m}}{\partial T}+U_0\frac{\partial v_{2m}}{\partial X}+u_{1m}\frac{\partial v_{1m}}{\partial X}+v_{1m}\frac{\partial v_{1m}}{\partial Y_m}=-\frac{\partial p_{2m}}{\partial Y_m},\quad \frac{\partial u_{2m}}{\partial X}+\frac{\partial v_{2m}}{\partial Y_m}=0,$$

we obtain

$$v_{2m}=-U_0\frac{\partial A_2}{\partial X}-\frac{\partial A_1}{\partial T}-\frac{dU_0}{dY_m}A_1\frac{\partial A_1}{\partial X}+U_0\int\limits_0^{Y_m}\frac{\partial p_{1m}}{\partial X}\left[\frac{1}{U_0^2}-\frac{1}{\lambda_1^2Y_m'^2(1+2\lambda_2\lambda_1^{-1}Y'_m)}\right]\times$$

$$\times dY'_m+U_0\int\limits_{\infty}^{Y_m}\frac{\partial p_{1m}}{\partial X}\frac{1}{\lambda_1^2Y_m'^2(1+2\lambda_2\lambda_1^{-1}Y'_m)}dY'_m$$

[as $A_1(T, X)$, the functions $A_2(T, X)$ are arbitrary].

Let us find the limiting form of expansions (1.2) near the wall. Using (1.1), at $Y_m \to 0$ we have

$$\begin{gathered} u=\lambda_1Y_m+\delta\lambda_1A_1+\ldots, \\ v=-\delta^{3/2}\lambda_1\frac{\partial A_1}{\partial X}Y_m-\delta^{5/2}\left(\frac{\partial A_1}{\partial T}+\lambda_1A_1\frac{\partial A_1}{\partial X}-\frac{\Delta}{\lambda_1}\frac{\partial^3A_1}{\partial X^3}\right)+\ldots, \\ p=p_\infty-\delta^2\Delta\frac{\partial^2A_1}{\partial X^2}+\ldots,\quad \Delta=\int\limits_0^{\infty}U_0^2dY_m. \end{gathered} \tag{1.4}$$

The first two terms of asymptotic series (1.4) for u and v become of the same order if $Y_m = O(\delta)$. This serves as justification for separately examining the wall region, where the new variable $Y_a = \delta^{-1}Y_m$ is on the order of unity. We seek the solution in the form

$$u=\delta u_a+\ldots,\quad v=\delta^{5/2}v_a+\ldots,\quad p=p_\infty+\delta^2p_a+\ldots \tag{1.5}$$

Substitution of Eqs. (1.5), with the functions u_a, v_a, and p_a dependent on T, X, and $Y_a = \mathrm{Re}^{1/2}\delta^{-1}y$, into the system of Navier–Stokes equations yields

$$\begin{gathered} \frac{\partial u_a}{\partial T}+u_a\frac{\partial u_a}{\partial X}+v_a\frac{\partial u_a}{\partial Y_a}=-\frac{\partial p_a}{\partial X}+O(\mathrm{Re}^{-1/2}\delta^{-7/2}), \\ \frac{\partial p_a}{\partial Y_a}=0,\quad \frac{\partial u_a}{\partial X}+\frac{\partial v_a}{\partial Y_a}=0. \end{gathered} \tag{1.6}$$

The symbol O in the conservation equation for the longitudinal component of momentum denotes the order of the viscous terms. At $\delta = O(\mathrm{Re}^{-1/7})$, the nonlinear boundary sublayer $Y_a = O(1)$ is viscous. This case was studied in [9, 10]. Below, we propose that $\delta \gg \mathrm{Re}^{-1/7}$. Then the nonlinear region, of the thickness $y = O(\mathrm{Re}^{-1/2}\delta)$, is in turn divided into an inviscid main part, where $Y_a = O(1)$ and the viscous terms in the first equation of system (1.6) are small, and a viscous sublayer directly adjacent to the surface in the flow, where $Y_a \sim (\mathrm{Re}^{-1/2}\delta^{-7/2})^{1/2} \ll 1$.

System (1.6), with the discarded viscous terms, must be augmented by the boundary condition of impermeability on the wall and limiting conditions on the upper edge of the nonlinear region. These conditions are given by Eqs. (1.4), rewritten relative to the variable Y_a. It is not hard to see that the functions

$$u_a = \lambda_1 Y_a + \lambda_1 A_1,$$
$$v_a = -\lambda_1 \frac{\partial A_1}{\partial X} Y_a - \frac{\partial A_1}{\partial T} - \lambda_1 A_1 \frac{\partial A_1}{\partial X} + \frac{\Delta}{\lambda_1} \frac{\partial^3 A_1}{\partial X^3}, \quad p_a = -\Delta \frac{\partial^2 A_1}{\partial X^2}, \tag{1.7}$$

ensuring satisfaction of the combining conditions at $Y_a \to \infty$, are the solution of nonlinear system (1.6). The impermeability condition remains to be satisfied. Let the source of the perturbations be the above-mentioned roughness on the surface in the flow. The form of this roughness is described by the equation $Y_a = G(T, X)$. Then the boundary condition has the form

$$v_a - \frac{\partial G}{\partial T} = \frac{\partial G}{\partial X} u_a \quad \text{at} \quad Y_a = G. \tag{1.8}$$

For the functions u_a and v_a given by Eqs. (1.7), condition (1.8) leads to the inhomogeneous Korteweg–de Vries equation

$$\frac{\partial A}{\partial t} + A\frac{\partial A}{\partial x} = \frac{\partial^3 A}{\partial x^3} - \varphi(t, x), \quad A = A_1 + G, \quad \varphi = \frac{\partial^3 G}{\partial x^3}, \tag{1.9}$$

in which we performed the transformation of the independent variables $T = \lambda_1^{-2}\Delta^{1/2}t$, $X = \lambda_1^{-1}\Delta^{1/2}x$ in order to eliminate the constants λ_1 and Δ. For normal injection from a plane surface $Y = 0$ with the velocity $v_a = v_n$, in Eq. (1.9) the inhomogeneous term $\varphi = \lambda_1^{-2}\Delta^{1/2}v_n$.

If the function A is found from unidimensional equation (1.9), then the two-dimensional velocity field and pressure are completely determined both in the nonlinear sublayer and in the main body of the jet.

2. Let us return to the solution of Eq. (1.9) and the Burgers equations obtained in [2]

$$\frac{\partial A}{\partial t} + A\frac{\partial A}{\partial x} = \frac{\partial^2 A}{\partial x^2} - \varphi(t, x) \tag{2.1}$$

as well as the Benjamin–Onoe equation [3, 4]

$$\frac{\partial A}{\partial t} + A\frac{\partial A}{\partial x} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\partial^2 A/\partial \xi^2}{\xi - x} d\xi - \varphi(t, x). \tag{2.2}$$

Given certain assumptions, Eqs. (2.1)-(2.2) reduce to the problem of the nonsteady interaction of inviscid perturbations in a boundary layer with a supersonic and subsonic gas flow, respectively. The function φ in the right sides of (2.1) and (2.2) is proportional to the rate of normal injection or is connected with the presence of an obstacle on the surface in the flow.

Equations (1.9), (2.1), and (2.2) describe the nonlinear evolution of perturbations with the wavelength λ, dependent on the amplitude δ and exceeding the characteristic transverse dimension of the flow of the order $Re^{-1/2}$. Meanwhile, the value $\delta \ll 1$ should be large compared to the scale of the amplitude in the asymptotic theory [6-10] where viscosity plays the deciding role. In particular, in regard to jet flow about a plane wall, the amplitude restriction is expressed by the inequality $\delta \gg Re^{-1/7}$.

Figure 1 ($t = 9.75$) shows the numerical solution of the Korteweg–de Vries equation (1.9) with zero initial data and an inhomogeneous term in the right side given by the expression $\varphi(t, x) = -2\sin(2\pi t)\sin(\pi x/5)$ in the rectangle $0 < x < 5$, $0 < t < 0.5$. Here $\varphi(t, x) = 0$ outside the rectangle. We thus modeled momentary suction from the wall jet through the slit. It can be seen from Fig. 1 that the local perturbation introduced is transformed into a wave packet propagating in the direction of an increase in x. The maximum peak is shifted downflow somewhat from the slit and decays slightly with time; new maxima and minima are generated in the leading part of the packet.

A different picture is seen when perturbations are continually introduced into the flow. This is evident from Figs. 2 and 3, which show the solutions of Eq. (1.9) for $\varphi = (\partial^2/\partial x^2)(x + \sqrt{x^2 + a^2})$, $a = 0.5$, and $A = 0$ at $t = 0$. The dashed curve in Fig. 2 shows the distribution of $-A$ at the moment of time $t = 4.35$. The curve indicates the formation of a wave packet which propagates downflow. In the region in which the external perturbations (near $x = 0$) described by the function φ are influential, the function $|A|$ gradually increases over time until the conditions for the isolation of a solitary wave are satisfied. The solid curve

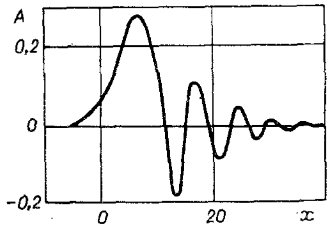

Fig. 1

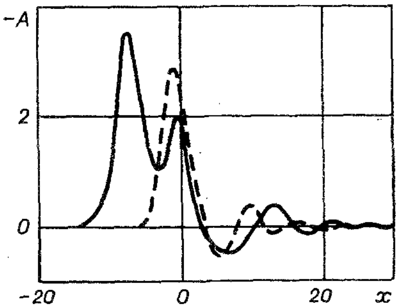

Fig. 2

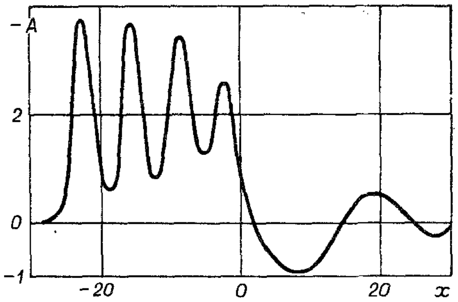

Fig. 3

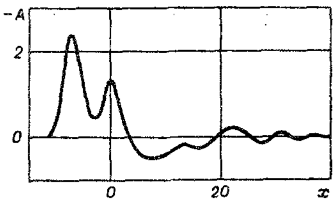

Fig. 4

corresponding to t = 8.9 illustrates the process of generation of the first such wave. Over time, solitary waves are periodically generated at the site of introduction of the perturbations and propagate in the form of a chain in the direction of negative x (Fig. 3, t = 21.7). The wave packet propagates in the direction of positive x with an increase in the characteristic period and a slight increase in amplitude.

As an example, for Eq. (2.2) we numerically solved a problem with zero Cauchy data on the assumption that $\varphi(t, x) = 1$ at $0 < x < 0.5$ and $\varphi(t, x) = 0$ outside this interval. Figure 4 shows the quantity $-A$ as a function of x for t = 22.6. The presence of the inhomogeneous term also leads to the formation of a group of solitary waves which propagate upflow from the perturbation region and a wave packet which moves downflow.

3. Steady-state inviscid equations describe the attached region of a supersonic flow [11] and flow in the neighborhood of boundary-layer separation on a surface moving downflow [12]. Two examples of nonsteady locally inviscid flows with an interaction obeying the Burgers equation were presented in [13]: supersonic flow about a plate with a periodically vibrating flap; and the propagation along the plane surface of a wedge of perturbations caused by a sudden change in the pressure gradient on its trailing edge.

We will examine the problem of the interaction of a boundary layer on a plate in a supersonic flow with a shock wave incident on the plate at the moment of time t = 0. We will assume that the pressure discontinuity, referred to the velocity head at infinity, is an order of magnitude greater than $Re^{-1/4}$ but remains small. Then Eq. (2.1), with the inhomogeneous term $\varphi = p_0\delta(x)$ (where $\delta(x)$ is the delta function and $p_0 > 0$) is valid in the region of the perturbed boundary layer. Here, p_0 is the dimensionless [6-8] pressure behind the shock wave with allowance for its reflection at the point x = 0. An inhomogeneous term of this type can also be interpreted as a consequence of deflection of the flap on the plate.

Assuming that A = 0 at t = 0, and performing the Cowle–Hopf transformation [14], $A = -2B^{-1}\partial B/\partial x$, we arrive at the linear problem

$$\frac{\partial B}{\partial t}=\frac{\partial^2 B}{\partial x^2}+\frac{p_0}{2}\theta(x)B,\ \ \theta(x)=\begin{cases}0, & x<0,\\ 1, & x>0,\end{cases}\ \ B(0, x)=B_0,\ \ \frac{dB_0}{dx}=0. \tag{3.1}$$

As a result of the Laplace transformation $\widetilde{B}(s, x)=\int_0^\infty B(t, x)\,e^{-st}dt$, it follows from (3.1) that

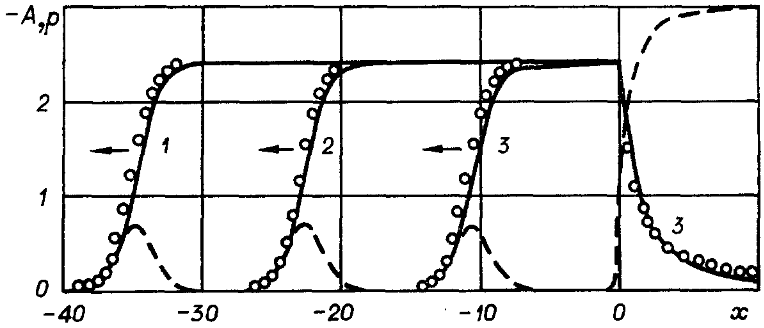

Fig. 5

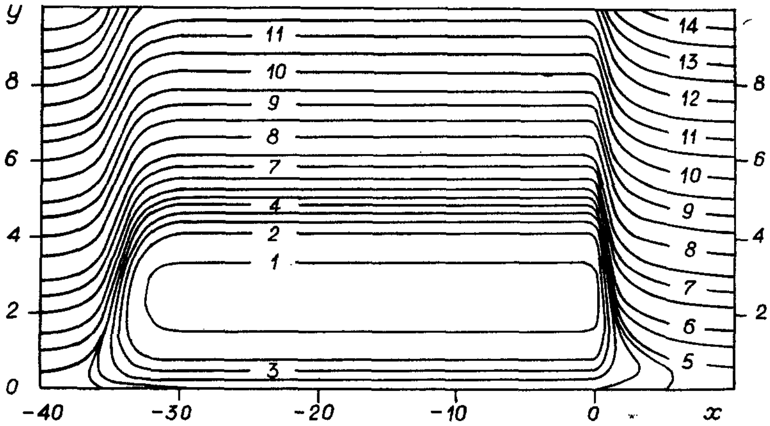

Fig. 6

$$\tilde{B}=\frac{B_0}{s-\frac{1}{2}p_0}+K_+\exp\left[-\left(s-\frac{1}{2}p_0\right)^{1/2}x\right],\quad x>0,\quad \tilde{B}=\frac{B_0}{s}+K_-\exp\left(s^{1/2}x\right),$$

$$x<0.$$

The coefficients K_- and K_+ are found from the requirement of continuity of the function $\tilde{B}$ and its first derivative at the point x = 0:

$$K_-=\frac{p_0B_0}{2s\left(s-\frac{1}{2}p_0\right)^{1/2}\left[\left(s-\frac{1}{2}p_0\right)^{1/2}+s^{1/2}\right]},\quad K_+=-\frac{s^{1/2}}{\left(s-\frac{1}{2}p_0\right)^{1/2}}K_-.$$

Changing over from the transforms to the originals, for x > 0 we find

$$B=B_0\exp\left(\frac{1}{2}p_0t\right)\operatorname{erf}\left(\frac{x}{2\sqrt{t}}\right)+B_0I(p_0,t),$$

$$I(p_0,t)=\frac{1}{\pi}\int_0^t\frac{1}{\sqrt{\tau(t-\tau)}}\exp\left(-\frac{x^2}{4\tau}+\frac{1}{2}p_0\tau\right)d\tau.$$

Similarly, for x < 0 we have the expression

$$B=B_0\operatorname{erf}\left(\frac{|x|}{2\sqrt{t}}\right)+B_0\exp\left(\frac{1}{2}p_0t\right)I(-p_0,t).$$

We reduce the convolution-type integral $I(p_0, t)$ to the form

$$I(p_0,t)=\frac{1}{\pi}\int_1^\infty\frac{1}{\mu(\mu-1)^{1/2}}\exp\left(-\frac{x^2}{4t}\mu+\frac{1}{2}p_0\frac{t}{\mu}\right)d\mu,\tag{3.2}$$

which is convenient for its evaluation at $t \to \infty$. The maximum of the function in the exponent of the integrand in (3.2) is reached at the lower boundary $\mu = 1$ of the interval of integration, while the maximum of this function in the interval $I(-p_0, t)$ is reached at the point $\mu = \sqrt{2p_0}t/|x|$. The use of the Laplace method to asymptotically evaluate the integral $I(p_0, t)$ over large times gives

$$A = -\frac{2}{x + (2/p_0)^{1/2}}, \quad x > 0. \tag{3.3}$$

The asymptote $I(-p_0, t)$ depends on $|x|/t$. With fixed x and $t \to \infty$, we have $A = -\sqrt{2p_0}$, $x < 0$. For finite values $\zeta = x + \sqrt{p_0/2}t$, the solution at $t \to \infty$ is expressed by the formula

$$A = -\sqrt{\frac{p_0}{2}}\left\{1 + \operatorname{th}\left[\frac{1}{2}\sqrt{\frac{p_0}{2}}\left(x + \sqrt{\frac{p_0}{2}}t\right) + \frac{1}{2}\ln\frac{2}{\sqrt{\pi p_0 t}}\right]\right\}, \quad x < 0. \tag{3.4}$$

It folows from (3.4) that a separation wave and a pressure pulse propagate upflow with a nondecaying amplitude and the phase velocity $D = -\sqrt{p_0/2} + O(t^{-1})$.

The problem of a sudden pressure drop behind the point $x = 0$ to $p_0 < 0$ is solved from the results obtained above by making the substitutions $A \to -A$, $x \to -x$, $t \to t$, $p_0 \to |p_0|$. In particular, a reduction in external pressure is accompanied by the downflow propagation of a wave with the phase velocity $D = \sqrt{|p_0|/2}$.

Solid lines 1-3 in Fig. 5 show the numerically constructed distributions of $-A(t, x)$ for $p_0 = 3$ at $t = 30$, 20, and 10. The distributions of total pressure $p(x) = p_0\theta(x) - \partial A/\partial x$ for the same times are shown by the dashed lines. In the region of the wave front, it is completely determined by the self-induced part $-\partial A/\partial x$, while near $x = 0$ the pressure is determined by the external and self-induced components. Located between them is a region with zero pressure. Over time, the size of this region increases in accordance with the motion of the wave. The circles show the function $-A(t, x)$ calculated from asymptotic formulas (3.3) and (3.4).

The function $A(t, x)$ found here can be used to construct the two-dimensional flow pattern. It is shown in Fig. 6 for the same calculation variant at $t = 30$. In this figure, lines 1-14 correspond to the values of the stream function -2.5, -1.5, -1.0, 0, 0.1, 1.0, 3.0, 6.0, 10, 15, 21, 28, 30, 45.

The solutions of the Burgers equation suggest that decay of the shock wave leads to the formation of a broad separation zone which extends upflow. Within the framework of the chosen model, the velocity of the leading edge of this zone is constant and is determined by the amplitude of the external perturbation. As regards the amplitude of such a separation wave, it remains constant over time and is unambiguously related to the rate of propagation upflow. Another action that would be just as effective in generating a separation wave in the boundary layer is steady slit injection of gas.

In studies which have involved numerical calculation of flow near an inflection point on a surface or a drop in external pressure on the basis of viscous equations of the theory of free interaction [6-8], investigators have noted either that the convergence of the finite-difference schemes deteriorates or that oscillatory regimes with an increase in the amplitude of the perturbing factor develop [15, 16]. It follows from the above that given sufficiently large angles of deflection of the surface or sufficiently high intensities of the incident shock wave, when inviscid equation (2.1) is valid, the problem has no steady-state solution.

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MEASUREMENT OF TURBULENCE CHARACTERISTICS IN COMPRESSIBLE BOUNDARY LAYERS NEAR SEPARATION ZONES

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Separated flows, distinguished by their great variety, are widely encountered in nature and in technology [1]. Until now, predicting their properties has been one of the most complex problems in fluid mechanics. Particular difficulties are encountered when analyzing turbulent separation due to the lack of a rigorous theoretical foundation. Most of the theoretical studies that have been conducted have involved the development of models of ideal liquids and gases and integral methods of jet and wake theory. Another focus has been the improvement of numerical methods of solving averaged Navier—Stokes equations with the use of semiempirical models of turbulence [2]. These directions of study have been taken in large part because of the available experimental data, which has been used to construct physical models of separated flows and to substantiate closing relations. In light of this, experiments now conducted in this field must necessarily be comprehensive in character.

The main difficulties encountered in experimentally studying compressible separated flows are related to measurements of turbulence characteristics in boundary layers. Such studies can be conducted on the basis of the use of laser-Doppler measurements of velocity or hot-wire anemometric instrumentation. Along with the familiar advantages and disadvantages of each method, the use of hot-wire anemometry allows the measurement of fluctuations of both gasdynamic and thermodynamic parameters. The presence of high-frequency pulsations of pressure, density, temperature, and velocity in a supersonic flow predetermines the requirements that must be met by hot-wire anemometric instruments and the measurement techniques. The possibility of broadly varying the temperature of the wire sensor T_W with a constant frequency range (which is necessary to separate pulsations of mass rate $\langle\rho u\rangle$ and stagnation temperature $\langle T_0\rangle$) is the main advantage of direct-current hot-wire anemometers (DCA) compared to fixed-resistance hot-wire anemometers (FRA) [4]. Another important advantage is that the DCA makes it possible to measure the internal noise of the instrument.

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